**Enrollment No: ______________________ Exam Seat No: ______________________**

# C.U.SHAH UNIVERSITY

## Summer Examination-2019

**Subject Name : Real Analysis**

**Subject Code : 4SC06RAC1** **Branch : B.Sc. (Mathematics)**

**Semester : 6** **Date : 16/04/2019** **Time : 10:30 To 1:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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**Q-1** **Attempt the following questions:** **(14)**

**a)** Define : Sequence . (1)
**b)** Write the lower bound of the sequence $\{ 1+\frac{(-1)^n}{n}\}$. (1)
**c)** What do you mean by a convergent sequence ? (1)
**d)** True/false: Every convergent sequence is bounded. (1)
**e)** Define : Series (1)
**f)** True/false: A series whose limit of $n^{th}$ term is zero need not be convergent. (1)
**g)** Define : limit point of sequence. (1)
**h)** True/false: Every Riemann integrable function have primitive. (1)
**i)** What do you mean primitive of function? (1)
**j)** Define: upper bound of sequence. (1)
**k)** What do you mean the partition of [a,b]? (1)
**l)** Find mess of partition P = $\{ -1 , 1.10 , 1.25, 2.50, 3 \}$ of [-1 ,3] (1)
**m)** True/false: $S(P, f)$ is always bounded. (1)
**n)** What do you mean by L(P ,f)? (1)

**Attempt any four questions from Q-2 to Q-8**

**Q-2** **Attempt all questions** **(14)**

**a)** Define : sequence of partial sum of series . **(2)**
**b)** State any one property of limsup and prove it. **(4)**
**c)** Find limsup and liminf of the following sequences. **(8)**
(1) $\{ 1 + \frac{n}{2^n} \}$ (2) $\{ 4 - 3(-1)^n \}$

**Q-3** **Attempt all questions** **(14)**

**a)** Define: Cauchy sequence **(2)**

**b)** Which of the following sequences are convergent ,divergent, oscillating finitely ,Oscillating infinitely ,bounded and unbounded ? **(4)**

(1) $\{ -2,-1, 0 , \frac{1}{2} , \frac{1}{4} , \frac{1}{8} \ldots\ldots\ldots\}$
(2) $\{n^3 + 1\}$

**c)** State and prove Cauchy's integral test for convergence of series. **(8)**

**Q-4** **Attempt all questions.** **(14)**

**a)** Define : p-series **(2)**

**b)** Show that the p-series is convergent if $p > 1$ and divergent if $p \leq 1$. **(5)**

**c)** State and Prove D'Alembert ratio test. **(7)**

**Q-5** **Attempt all questions.** **(14)**

**a)** State Cauchy's root test. **(2)**

**b)** Show that the series is convergent iff it's sequence of partial sum is convergent. **(4)**

**c)** Test the convergence of the following series. **(8)**

(1) $\frac{1}{5} + \frac{1}{2.5^2} + \frac{1}{3.5^3} + \ldots\ldots$ (2) $\sum \frac{1}{n^{\frac{5}{4}}+1}$

(2) $\sum \frac{n^2}{3n^2+4}$ (4) $\sum \frac{9^n}{10^n+3}$

**Q-6** **Attempt all questions.** **(14)**

**a)** What is alternating series? State and prove Leibnitz test for alternating series . **(7)**

**b)** Explain the following terms: **(4)**

(1) Absolutely convergent series.
(2) Conditional convergent series.

**c)** Show that the series $\sum \frac{(-1)^{n+1} \ n^2}{n^3+4}$ is convergent. **(3)**

**Q-7** **Attempt all questions.** **(14)**

**a)** Define : Riemann integrable function. **(2)**

**b)** Show that sum of two Riemann integrable function on [a , b] is also Riemann integrable. **(7)**

**c)** Give an example of function which is not Riemann integrable justify it . **(5)**

**Q-8** **Attempt all questions.** **(14)**

**a)** Define: Riemann upper sum of the function f(x) on [a ,b]. **(2)**

**b)** Show that every monotonic function on [ a , b] is Riemann integrable . **(6)**

**c)** State and prove fundamental theorem of calculus. **(6)**